Quantum Theory of the Laser

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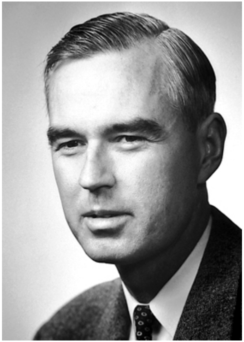

Figure: Left: Hermann Haken. Right: Willis Lamb

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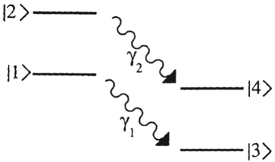

Figure: Four-level atomic model of a laser.

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- Then the density operator can be written as $\rho = \sum_{n,m=0}^{\infty} \rho_{n,m}(0)|n\rangle\langle m|$

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- Master equation for this system that we have described:

$$\begin{aligned}\frac{d\rho}{dt} &= ig[a^\dagger\sigma_-^{12} + a\sigma_+^{12}, \rho] \\ &+ \frac{\gamma_1}{2}(2\sigma_-^{13}\rho\sigma_+^{13} - \sigma_+^{13}\sigma_-^{13}\rho - \rho\sigma_+^{13}\sigma_-^{13}) \\ &+ \frac{\gamma_2}{2}(2\sigma_-^{24}\rho\sigma_+^{24} - \sigma_+^{24}\sigma_-^{24}\rho - \rho\sigma_+^{24}\sigma_-^{24})\end{aligned} \tag{1}$$

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- Fokker-Planck is just the time evolution of the probability density function
- That is, we model the system as a stochastic process (e.g. Brownian motion)

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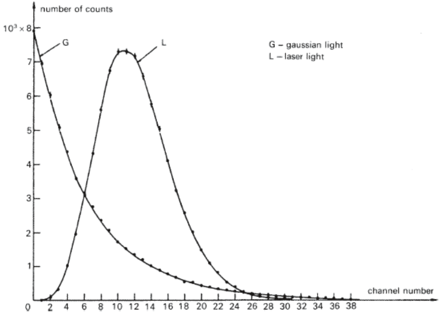

Figure: Poisson Distribution for Lasers

- Note that this is for lasers above the threshold (i.e., the gain coefficient is greater than the cavity decay rate)

References

- Walls, D.F. and Milburn, Gerard J. *Quantum Optics*. Springer, Verlag, 2008.
- Haken, Hermann. *Laser Theory*. Springer, Verlag, 1984.
- Various Wikipedia entries