Measuring the Balmer Series and Rydberg Constant Using Visible-Light Spectrometer

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Abstract

The hydrogen atom plays an important role in the theories of quantum mechanics. Spectral lines of hydrogen exhibit the transitioning electrons between energy levels which are quantized. These spectra make up series in various regions of the electromagnetic spectrum. The wavelengths of these series can be calculated using the Rydberg equation. One such series, the Balmer series, is the spectra located in the visible region of light. The Balmer series and Rydberg's equation are of great importance as they allow for a glimpse into the realm of quantum mechanics and astronomy.

In this paper, a visible-light spectrometer is used to measure and exhibit the effects of the Balmer series in the hydrogen spectrum. Using *Logger Pro* software along with a spectrometer and hydrogen gas-filled tube, we can show the simplicity and precision of observing spectral line emissions of the hydrogen atom. Using this equipment, our results demonstrate how the wavelengths of the Balmer series can be

manipulated to find Rydberg's constant. Numerically, we calculated Rydberg's constant to be $R = 1.0973 \times 10^7 \pm 2.01 \times 10^3\ m^{-1}$ consistent within two sigmas of the accepted value of $R = 1.097 \times 10^7\ m^{-1}$.

Introduction

Among the many facts about hydrogen, such as its abundance and properties, the most notable are those that pertain to quantum and atomic theory. In this paper, we will focus on the spectrum of hydrogen, or the wavelengths of light emitted by an element when they are excited electrically. More specifically, on the Balmer series, the visible wavelengths of hydrogen spectral lines.

Johann Balmer was the first to discover such a series which he derived into an equation that fit his data [1]. However, hydrogen has spectral lines with other energies, undetected in visible light. These other spectral lines depend on the final energy level of the transitioned electrons. Johannes Rydberg found a relation that describes all wavelengths of the spectral lines of hydrogen [2]. Rydberg's equation was, for a time, purely empirical until Niels Bohr created his famous Bohr model for the hydrogen atom. In his massive paper, Bohr was able to build a framework theorizing that the spectral lines of hydrogen are a direct result of transitioning electrons between energy levels, producing Rydberg's equation [3].

In this paper, we will observe the Balmer series and determine Rydberg's constant from collected data. We will do this by using collection software, a spectrometer and hydrogen gas-filled tube to calculate wavelengths of the Balmer series.

Experimental Method

In this experiment, we will use a spectrometer, hydrogen discharge tube and computer software to measure visible wavelengths emitted by hydrogen. The experiment configuration is shown in Figure 1. The spectrometer is used to measure the wavelengths of the light being emitted from the hydrogen discharge tube. It does this by diffracting the light it receives via a sensor. It records these diffracted wavelengths and generates values in *Logger Pro*. The hydrogen discharge tube emits a spectrum of light by applying a voltage to a tube filled with hydrogen gas. The current excites the electrons which allow them to move to higher energy levels and then emit light by transitioning down to a lower energy level. The final energy level of the electrons is the same for wavelengths in certain regions of light. These wavelengths make up series of spectral lines, such as the Balmer series.

Attaining results required careful positioning of the discharge tube and the spectrometer. To reduce interference from other light sources, the discharge tube and spectrometer are positioned close together. This optimizes the amount of light the spectrometer receives from the discharge tube.

Using *Logger Pro*, we can measure these wavelengths detected by the spectrometer via fiber-optic cable. *Logger Pro* allows for the selection of peak wavelengths recorded by the spectrometer. These wavelengths appear in four different regions of visible light and are distinct from the rest of the spectra. Repeating the experiment allows for refinement and precision of data. We can easily run the experiment several times to get very precise values for the peak wavelengths. Doing this exemplifies the simplicity and reciprocity of measuring visible wavelengths using a visible-light spectrometer. Using these wavelengths, we can find Rydberg's constant using fit parameters.

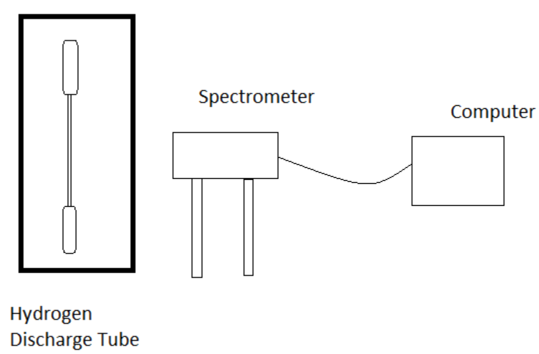

Figure 1. Experiment configuration set-up. Shown from left to right is a hydrogen discharge tube used to emit the light and a visible-light detecting spectrometer connected to a computer with *Logger Pro* software.

Theoretical Background

The equation first used to describe the Balmer series is given by

$$\lambda = \frac{hm^2}{m^2 - n^2}, \tag{1}$$

where n = 2 (unbeknownst to Balmer, n is the energy level), h is Balmer's constant and m can attain values of integers starting at m = 3 [1].

Rydberg generalized this to describe all spectral series of hydrogen given by

$$\frac{1}{\lambda} = R(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}), \tag{2}$$

where λ is the wavelength, R is Rydberg's constant, n_f is the final energy level of the transitioning electrons and n_i is the initial energy level [2].

The Rydberg equation was empirical until Niels Bohr discovered what came to be known as the Bohr model [3]. We start with energy given by Bohr,

$$E_r = \frac{2\pi^2 me^4}{n^2h^2}, \tag{3}$$

where E_r is the radiated or absorbed energy, m is the mass of the electron, $e = 1.602 \times 10^{-19}\ C$, $\pi \cong 3.14159$, n is the energy level and $h = 6.602 \times 10^{-34} J \cdot s$, Planck's constant. This energy is the binding energy, the energy associated with each energy level of an atom. This energy demonstrates how energy is quantized and not continuous.

Then we can take Equation 1 and find its change in energy when transition states are considered,

$$\Delta E_r = E_{r_f} - E_{r_i} = \frac{2\pi^2 me^4}{h^2} \cdot \left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right) = 13.6\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right), \tag{4}$$

where ΔE_r is the change in radiated or absorbed energy between energy levels, n_f is the final energy level and n_i is the initial energy level. Note that n_f will not change for the spectral series one analyzes ($n_f = 2$ for Balmer, etc.). Equation 2 shows that when an electron changes energy levels, it will radiate energy in the form of electromagnetic radiation.

Then using the equation for photon energy,

$$\Delta E = \frac{hc}{\lambda}, \tag{5}$$

we can solve for $\frac{1}{\lambda}$ to get,

$$\frac{1}{\lambda} = R(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}), \tag{6}$$

where R is 13.6/hc (Rydberg's constant). Notice that Eq. 6 takes the form of a line. The energy of a photon is another result of quantized energy, a consequence of the photoelectric effect.

Analysis

From the data collected, we were able to find the average wavelength of the visible Hydrogen spectra emission lines at each energy level shown in Table I. Using the average wavelength, we plot $\frac{10^3}{\lambda_{avg}}\ vs.\ (\frac{1}{4} - \frac{1}{n^2}) \times 10$. We set the intercept to zero to get a $y = mx$ fit and match Eq. 6, as shown in Figure 2. We found the slope of this plot to be

$m = 1.0973 \pm 2.01 \times 10^{-4}\ \mu m^{-1}$. The error in the slope was found using the method of least squares (Excel function *Linest*).

From the slope, we can calculate Rydberg's constant. Since our plot is fitted to y = mx, we can see that this matches Rydberg's equation (Eq. 6). We can then see that our slope is Rydberg's constant with a factor difference of 10^7. This is because our slope is in micrometers. Converting this to meters, the value for Rydberg's constant we calculated is $R = 1.0973 \times 10^7 \pm 2.01 \times 10^3\ m^{-1}$. The standard error was calculated using the slope's standard error calculated using least squares and refactored by 10^7.

Line	Color	λ (nm) Run #1	λ (nm) Run #2	λ (nm) Run #3	Average wavelength (nm)
Hα, n = 3	Red	658	656	656	657
Hβ, n = 4	Blue-green	486	486	486	486
Hγ, n = 5	Blue	434	434	434	434
Hα, n = 6	Violet	410	410	410	410

Table I. Balmer series emission lines. Table for each initial energy level n and average wavelength.

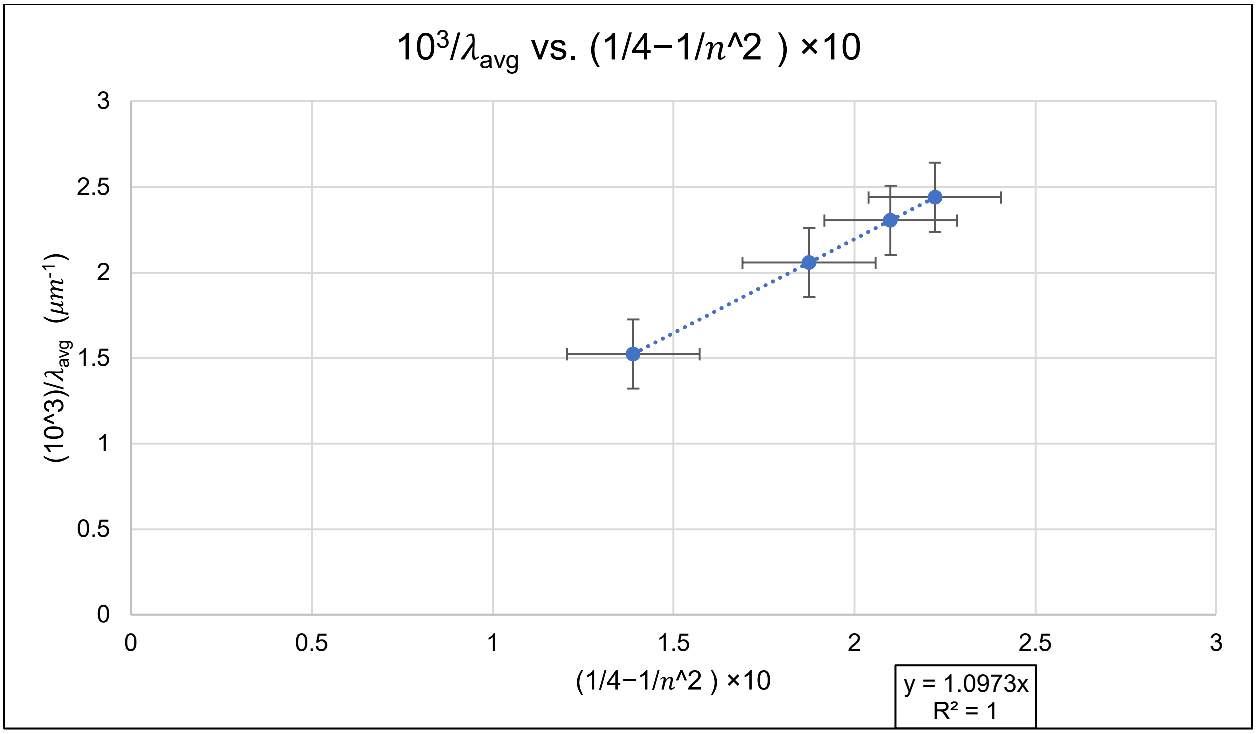

Figure 2. Plot of inverse average wavelength versus inverse change in transition states. Note that the intercept was set to zero to fit the plot to fit Eq. 6. The inverse average wavelength ($\frac{10^3}{\lambda_{avg}}$) plotted on the y-axis is measured in μm^{-1} and the change in transition states plotted on the x-axis is unitless.

Conclusion

In this paper, we presented experimental techniques to measure the Balmer series line emissions and found Rydberg's constant using a visible-light spectrometer. We investigated the visible wavelengths in the Balmer series and plotted them to visualize the Rydberg relation. We accomplished this using a hydrogen discharge tube illuminated against a spectrometer. Using the spectrometer, we were able to measure peak wavelengths from a visible spectrum of light. Our results allow us to calculate Rydberg's constant. We found this value to be $R = 1.0973 \times 10^{7} \pm 2.01 \times 10^{3}\ m^{-1}$. This value is consistent and within two sigmas of the accepted value of Rydberg's constant, R = $1.097 * 10^{7}\ m^{-1}$.

The errors in this paper were significantly reduced from earlier experiments performed by the like of Anders Jonas Ångström, who first found the visible wavelengths that was later named the Balmer series (after Balmer who found the relation) [4]. The simplicity of our methods provides us this luxury. However, there was error associated with the experiment. For instance, there may have been unaccounted interference from light in the environment. To reduce this, we could repeat the experiment in a dark room. Repeating the experiment in a dark room would allow us to verify if there is interference from external visible light and give us a numerical value for the error.

The hydrogen spectrum is important in physics, as it helped Bohr and other scientists develop atomic and quantum theories. The hydrogen spectrum also presents a use to astronomers since it allows them to find stellar objects with large abundances of hydrogen. One such application could be employed to detect black holes by measuring the hydrogen in accretion disks that are around them [5]. Rydberg's constant is just one example of the implications of the hydrogen atom and its many uses in physics.

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